Example 7.2-1: View Factor with the Crossed and Uncrossed String Method

A long beam with a triangular cavity (surface 1) has the dimensions shown in Figure 1. A 0.75 m wide plate (surface 2) is positioned parallel to the beam at a distance 0.5 m below it. The surroundings are taken to be surface 3. Determine all of the view factors for surfaces 1 and 2 to each other and to surface 3.
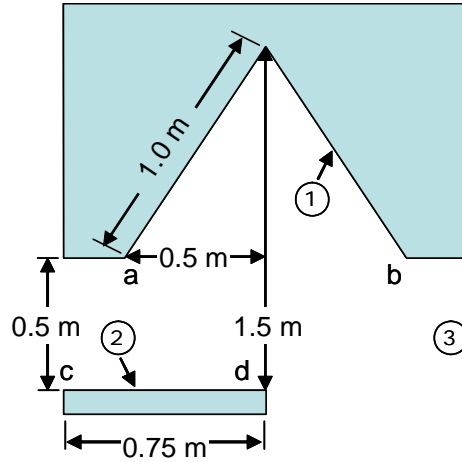

Figure 1: Long beam with triangular cavity

Since the beam and plate extend for an indeterminate distance (W) into the paper, the geometry is two-dimensional. The simplest way to determine the view factors is to use the crossed and uncrossed string method, as indicated in Eq. **Error! Reference source not found.**.

The area of the triangular cavity, A_1, is (2 m) x W where W is set to 1 m since it cancels out of the geometric calculations. The area of the plate, A_2, is 0.75 m^2. Since the top surface of the plate cannot see itself, $F_{2,2} = 0$. The view factor between surface 1 and 2 is found by applying Eq. **Error! Reference source not found.**:

$$A_1 F_{1,2} = \frac{\left(\overline{ad} + \overline{bc}\right) - \left(\overline{ac} + \overline{bd}\right)}{2} \tag{1}$$

The lengths in Eq. (1) are found from trigonometry:

$$\overline{ad} = \sqrt{\left(0.5 \text{ m}\right)^2 + \left(0.5 \text{ m}\right)^2} = 0.707 \text{ m}$$

$$\overline{bc} = \sqrt{\left(0.5 \text{ m}\right)^2 + \left(1.25 \text{ m}\right)^2} = 1.346 \text{ m}$$

$$\overline{ac} = \sqrt{\left(0.5 \text{ m}\right)^2 + \left(0.25 \text{ m}\right)^2} = 0.559 \text{ m}$$

$$\overline{bd} = \sqrt{\left(0.5 \text{ m}\right)^2 + \left(0.5 \text{ m}\right)^2} = 0.707 \text{ m}$$

Substituting these lengths into Eq. (1) leads to $F_{1-2} = 0.197$. The view factor of surface 1 to itself, F_{1-1}, can be found in several ways. Perhaps the easiest way is to determine F_{1-1} is to place an imaginary surface, call it surface 4, between points a and b; note that $A_4 = 1$ m^2. The view factor of surface 4 to surface 1 is 1.0 because surface 4 cannot see itself. By reciprocity $A_4 F_{4-1} = A_1 F_{1-4}$ so $F_{1-4} = 0.5$. Surface 1 can only see itself and surface 4 so, according to Eq. **Error! Reference source not found.**,

$$F_{1,1} = 1 - F_{1,4} = 0.5$$

With F_{1-1} and F_{1-2} now known, Eq. **Error! Reference source not found.** can be used to determine F_{1-3}:

$$F_{1,3} = 1 - F_{1,1} - F_{1,2}$$

The remaining view factors can be found by applying reciprocity.

$$F_{2,1} = A_1 F_{1,2} / A_2 = 0.3936$$
$$F_{2,2} = 0$$
$$F_{2,3} = 1 - F_{2,1} - F_{2,2} = 0.6064$$