EXAMPLE 2.8-2: Resistance of a Square Channel

Figure 1 illustrates a square channel. The inner and outer surfaces are held at different temperatures, T_1 = 250°C and T_2 = 50°C, respectively.

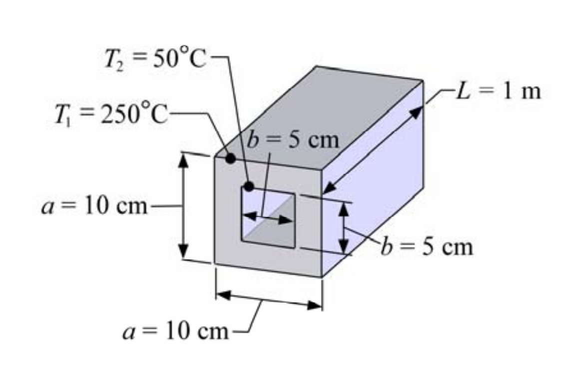

Figure 1: Square channel with heat transfer from inner to outer surface

The outer dimension of the square channel is a = 10 cm and the inner dimension is b = 5.0 cm. The thermal conductivity of the material is k = 100 W/m-K and the length of the channel is L = 1 m.

a.) Using an appropriate shape factor, determine the actual rate of heat transfer through the square channel.

The inputs are entered in EES:

```
"EXAMPLE 2.8-2: Resistance of a Square Channel"
$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

"Inputs"
a=10[cm]*convert(cm,m)             "outer dimension of square channel"
b=5[cm]*convert(cm,m)              "inner dimension"
L=1[m]                             "length"
k=100[W/m-K]                       "material conductivity"
T_1=converttemp(C,K,250 [C])       "inner wall temperature"
T_2=converttemp(C,K,50 [C])        "outer wall temperature"
```

This problem is a 2-D conduction problem; however, the solution for this 2-D problem has been developed and is correlated in the form of a shape factor, S. Shape factors are discussed in Section 2.1. The shape factor (S) is defined according to:

$$\dot{q}_{cond} = S\,k\left(T_1 - T_2\right)$$

where $\dot{q}_{cond}$ is the rate of conductive heat transfer between the two surfaces at T_1 and T_2. The particular shape factor for a square channel can be accessed from EES' built-in shape factor library. Select Function Info from the Options menu and select Shape Factors from the lower-right pull down menu. Use the scroll-bar to select the shape factor function for a square channel.

The function SF_7 is pasted into the Equation Window using the Paste button and used to calculate the actual heat transfer rate:

```
"Actual heat transfer rate"
SF=SF_7(b,a,L)                    "shape factor for square channel"
q_dot=SF*k*(T_1-T_2)              "heat transfer"
q_dot_kW=q_dot*convert(W,kW)      "heat transfer in kW"
```

The actual heat transfer rate predicted using a shape factor solution is 211.4 kW.

b.) Provide a lower bound on the heat transfer through the square channel using an appropriate 1-D model.

According to Eq. (2-237), the adiabatic or average length models can be used to provide an upper bound on the resistance of the square channel. The adiabatic model does not allow the heat to spread as it moves across the channel, as shown in Figure 2(a).

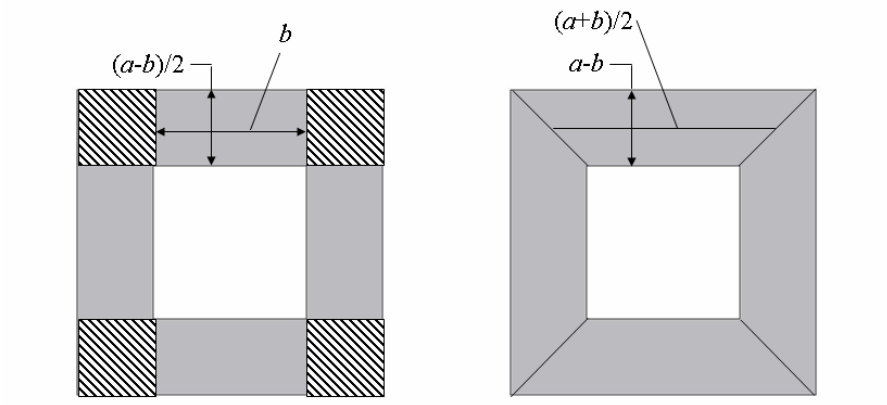

Figure 2: 1-D models based on the (a) adiabatic resistance allows no heat spreading and (b) the average area uses the average area along the heat transfer path.

The resistance in the adiabatic limit is equal to the resistance of a plane wall with an area equal to the internal surface area of the channel and length equal to the channel thickness:

$$R_{ad} = \frac{(a-b)}{8\,k\,L\,b}$$

The rate of heat transfer predicted by the adiabatic limit is:

$$\dot{q}_{ad} = \frac{(T_1 - T_2)}{R_{ad}}$$

```
"Adiabatic limit"
R_ad=(a-b)/(8*k*L*b)           "thermal resistance in the adiabatic limit"
q_dot_ad=(T_1-T_2)/R_ad        "heat transfer in the adiabatic limit"
q_dot_ad_kW=q_dot_ad*convert(W,kW)
```

The adiabatic model predicts 160.0 kW and therefore leads to a 32% underestimate of the actual heat transfer rate (211.4 kW from part (a)).

c.) Provide an upper bound on the heat transfer rate using an appropriate 1-D model.

Equation (2-237) indicates that either the isothermal or average area approach can be used to establish a lower bound on the thermal resistance and therefore an upper bound on the heat transfer. The average area along the heat transfer path is shown in Figure 2(b). The resistance calculated according to the average area model is:

$$R_{\bar{A}} = \frac{(a-b)}{4\left(a+b\right)L\,k}$$

The heat transfer in this limit is:

$$\dot{q}_{\bar{A}} = \frac{\left(T_1 - T_2\right)}{R_{\bar{A}}}$$

```
"Average area limit"
R_A_bar=(a-b)/(4*(a+b)*k*L)          "thermal resistance in the average area limit"
q_dot_A_bar=(T_1-T_2)/R_A_bar        "heat transfer in the average area limit"
q_dot_A_bar_kW=q_dot_A_bar*convert(W,kW)
```

The average area approximation predicts a heat transfer rate of 240 kW and is therefore a 12% overestimate of the actual heat transfer rate.