Introduction to Statics

1.1E Getting Started with EES

EES (pronounced 'ease') is an acronym for Engineering Equations Solver. The basic function provided by EES is the numerical solution of equations. This capability allows the engineer or student to focus on the underlying problem that is being considered rather than the often tedious math that is required to solve it. EES allows more complex and interesting problems to be considered and reduces the possibility of making errors. Unit checking and unit conversions are built into EES and the software can quickly carry out parametric studies and prepare high quality plots. All of these features make EES a powerful tool for developing mathematical models of statics problems. This chapter discusses the basic features of EES and ends with examples and problems that apply EES to problems related to Chapter 1 of the text. Videos that support the lessons throughout this book are linked directly in the pdf.

Installing EES

There are several types of licenses available and the one you have access to depends on your institution. These range from student single user licenses to commercial and professional single and multi-user licenses, all the way to commercial and professional academic licenses. All of these licenses will work on any modern 32 and 64-bit Windows operating systems.

To install EES on your computer you will need both Setup_EES.exe and a license (.dft) file. These files should have been provided to you when you purchased the program or by a system administrator if you obtained the program from your company or institution. Execute the Setup_EES.exe program and proceed until you see the Select Destination Directory dialog. The default directory that EES will be installed into is named EES32. You can change the directory name, if you wish, but remember where you installed it because you will need to copy the license file into the EES installation direction. The final window, Figure 1.1, provides several installation options. If selected, the Install Heat Transfer Library option will install an extensive set of library functions that facilitate heat transfer calculations. Unless you have customized your units or constants files then you will want to overwrite these files.

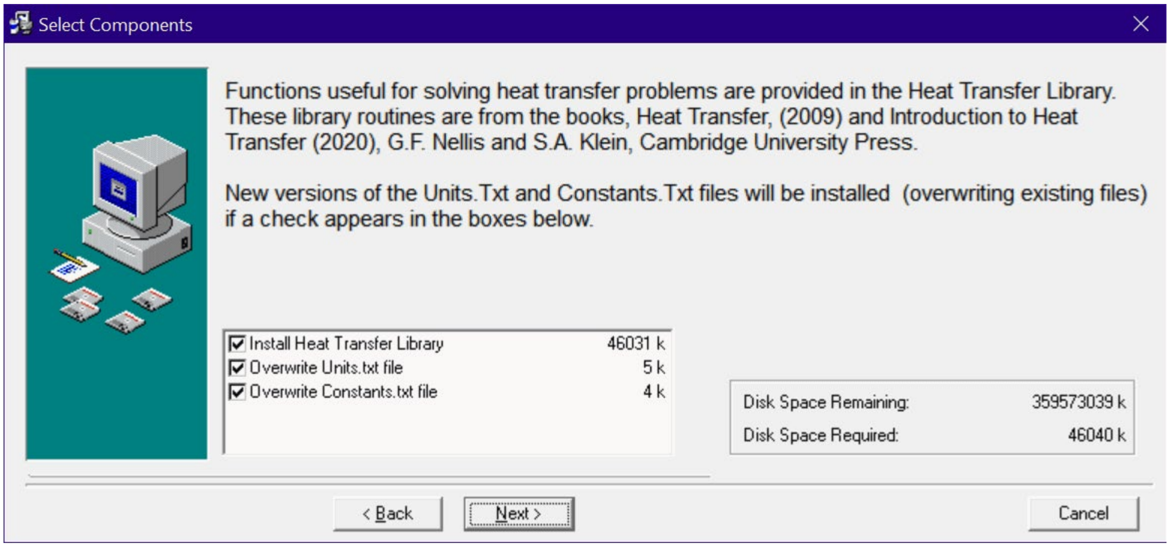


Figure 1.1: Select Components dialog.

Clicking the Next button will install the program. If EES fails to start, the most likely problem is that the license file is not contained in the directory that the program was installed in. In this case, copy the license (.dft) file into the install directory and then restart EES by double-clicking on the EES.exe icon or by entering C:\EES32\EES.exe in the Windows run dialog.

Entering and Solving Equations

When you open EES for the first time you will see the Equations Window. The Equations Window is where the equations to be solved are entered and you can return to it at any time by selecting Equations from the Windows menu or by using the shortcut Ctrl+E. EES allows the user to enter equations rather than assignments, as are required by most formal programming languages. In an assignment statement, the value of each variable on the right side of an expression must have been previously determined. A set of equations is simply a mathematical relationship between variables and therefore does not have this restriction.

For example, consider the set of equations:

$$x + y = z^3 \tag{1.1}$$

$$y = z - 4 \tag{1.2}$$

$$z = x^2 - 3 \tag{1.3}$$

Equations (1.1) through (1.3) are three non-linear equations in the three unknowns *x*, *y*, and *z*. These equations are not directly solvable using most formal programming languages because they are equations rather than assignments. At its most fundamental level, EES is an equation solver which solves sets of nonlinear equations like these directly. We can enter these equations in the Equations Window as shown in Figure 1.1.

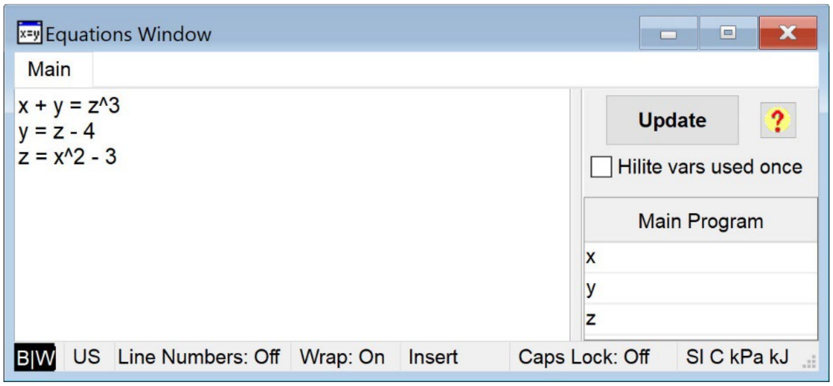


Figure 1.2. Equations Window with Equations (1.1) through (1.3) entered.

In this book, text that is entered in the Equations Window will be indicated by being shaded, as shown below.

```
x + y = z^3
y = z - 4
z = x^2 - 3
```

Select Solve from the Calculate menu (or use the shortcut F2) in order to initiate the iterative process that EES uses internally to solve the system of equations. EES will re-order and block the equations in a logical manner and then, starting from a guessed solution it will iteratively search for an actual solution to the equations. The result should be the dialog shown in Figure 1.2 which shows that the calculations have been completed and provides some of the details of the process.

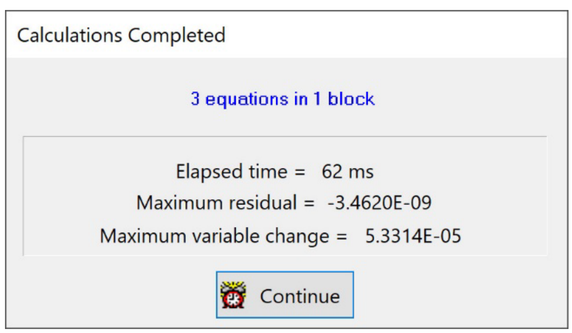


Figure 1.3. Dialog indicating that calculations have been successfully completed.

Select Continue to proceed to the Solutions Window, which is shown in Figure 1.3[1].

[1] Note that this book assumes that the US numerical format is in use which corresponds to a decimal point for the decimal separator, comma for a list separator, and semicolon for equation separator. To switch to the EU numerical format please see this link.

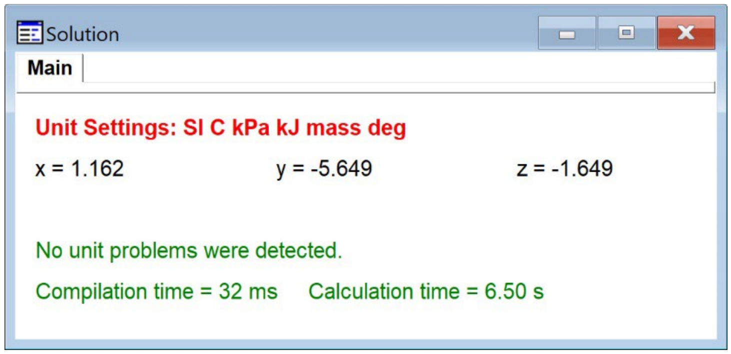


Figure 1.4. Solutions Window shows the values of each of the variables involved in the problem.

EES has identified one of the solutions to the equation set entered in the Equations Window. The process of solving the equations was taken care of internally; the user is not required to carry out any algebra or iteration.

Let's return to the Equations Window. The equations can be entered in any order and rearranged algebraically in any way. For example, we can switch the order of the first two equations and switch the right and left sides of the third equation and EES will identify the same solution.

```
y = z - 4
x + y = z^3
x^2 - 3 = z
```

The Equations Window functions a lot like a word processor. You can use the cut (Ctrl+X), copy (Ctrl+C), paste (Ctrl+V), and undo (Ctrl-Z) commands just as you would in Microsoft Word® or other programs that manipulate text. Equations are typically entered one per line; however, multiple equations can be entered on the same line provided that they are separated by a semicolon.

The mathematical operators and order of operation used in the equations are consistent with those used by most any programming language. Variable names in EES must start with a letter but they can include any U.S. keyboard character except ( ) * / + - { } or ;. EES is case-insensitive; that is, the symbols x and X are interpreted to be the same variable in the Equations Window.

EXAMPLE E1.1 *Settling the Bill*

Three friends have gone to dinner together and it's time to settle the bill. The total bill is $128.55 but it is not itemized. The bill includes 5% sales tax and 20% gratuity. Holly had a steak dinner. Debbie's meal was 75% the cost Holly's steak dinner and Tom's meal cost twice that of Debbie's. Determine how much each person's meal cost.

SOLUTION

Road Map This is an easy problem meant to illustrate how to enter and solve a small set of equations using EES. We will develop an equation corresponding to each of the statements in the problem and then verify that we have an equal number of unknowns as we do equations.

Governing Equations The cost of each of the meals (without tax and gratuity) will be indicated by the variables *Debbie*, *Tom*, and *Holly*. The problem provides relationships between the cost of these meals:

$$Debbie = 0.75\,Holly, \tag{1}$$

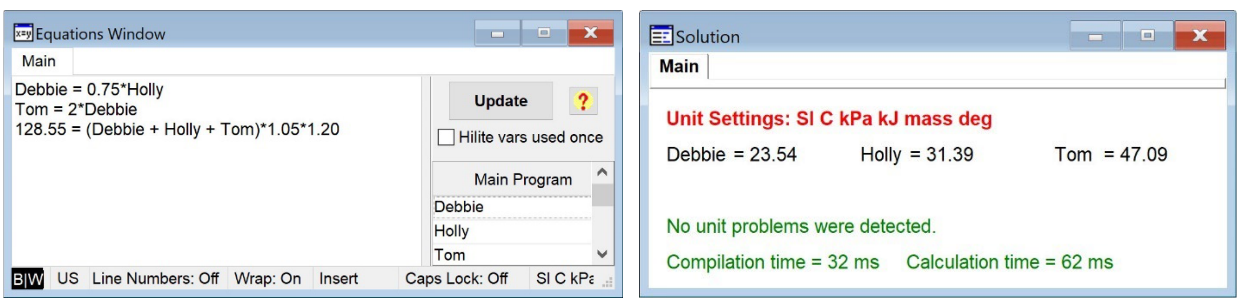
$$Tom = 2\,Debbie, \tag{2}$$

as well as the cost of the entire bill, including tax and gratuity. We will assume that the gratuity is applied to the after tax cost of the meals:

$$128.55 = \left(Debbie + Holly + Tom\right)\left(1.05\right)\left(1.20\right). \tag{3}$$

Equations (1) through (3) are three equations in the three unknowns *Debbie*, *Tom*, and *Holly*.

Computation Equations (1) through (3) are entered in the Equations Window of EES, as shown in Figure 1(a). Selecting Solve from the Calculate menu (or hitting the Shortcut Key F2) provides the solution in the Solution Window, shown in Figure 1(b).



(a) (b)

Figure 1. (a) Equations Window and (b) Solution Window.

Discussion & Verification The solution to Eqs. (1) through (3) by hand would have been tedious and provided an opportunity to make mathematical errors.

Comments and Key Variables

It is good practice to annotate any computer code in order to clarify the meaning behind each line. Typically, each equation will be followed by a comment that is ignored by the equation solver itself but is visible to the user. Comments in the Equations Window should be enclosed either in curly braces or quotes. Any line that begins with two forward slashes is also considered a comment. The comments will be displayed in blue (by default) in the Equations Window to indicate that they are not part of the equation set. For example, Figure 1.4 shows the Equations Window from Example 1.1 with comments added after each line.

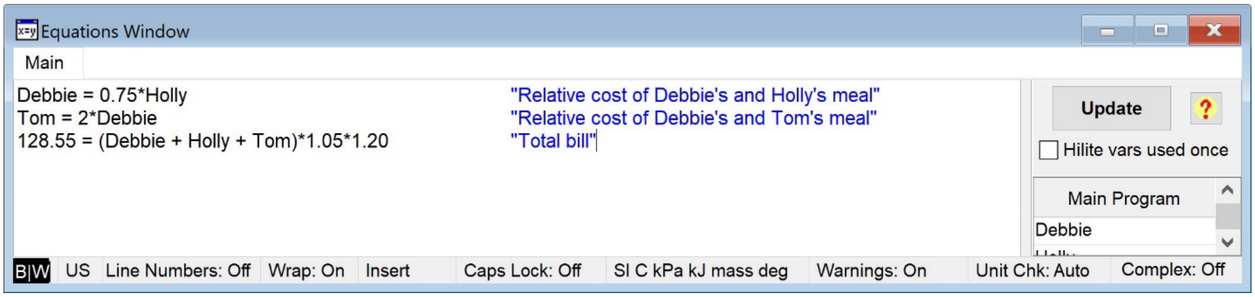
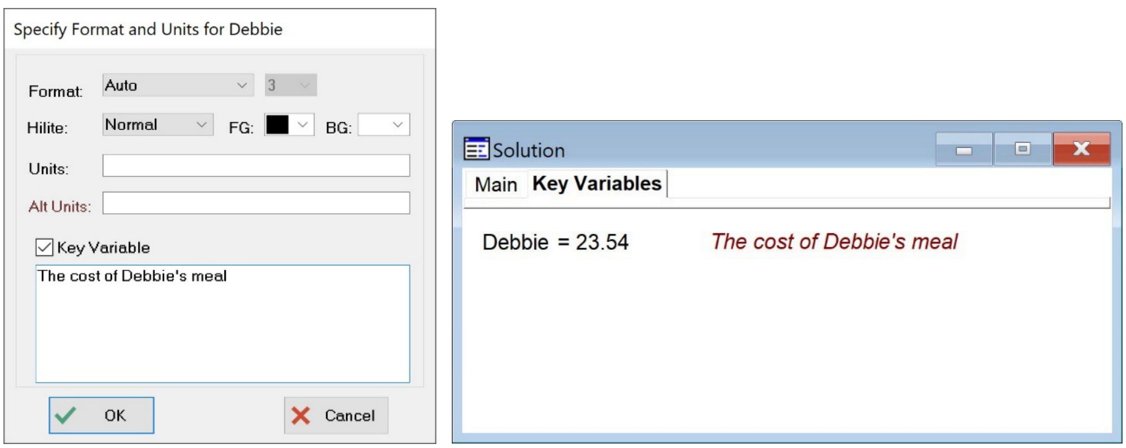


Figure 1.5. Equations Window from Example 1.1 with comments.

The Solution Window in a large EES program may contain many variables making it difficult to quickly and easily identify the small subset of variables that are of primary interest. You can select a subset of the variables in your Solution Window and make them key variables. Right click on the variable of interest in the Solutions Window to bring up the dialog shown in Figure 1.5(a). Selecting the Key Variable check box will make an edit box appear in which you can write some descriptive comments about the variable. Select OK and you will see that a new tab will appear in the Solutions Window allowing access to the Key Variables Window, as shown in Figure 1.5(b).



(a) (b)

Figure 1.6. (a) Specify Format and Units dialog for the variable *Debbie*. Note that the key variables check box has been checked and a description entered. (b) The Key Variables tab in the Solutions Window.

Your instructor may require that you make the final answers to each section of a homework problem into a key variable so that it is easy to grade.

Printing Your Solution

Once you have completed your EES solution, you will need to print either a hard copy or a pdf of your file to an installed printer. To do this, select Print from the File menu to bring up the Print dialog shown in Figure 1.6.

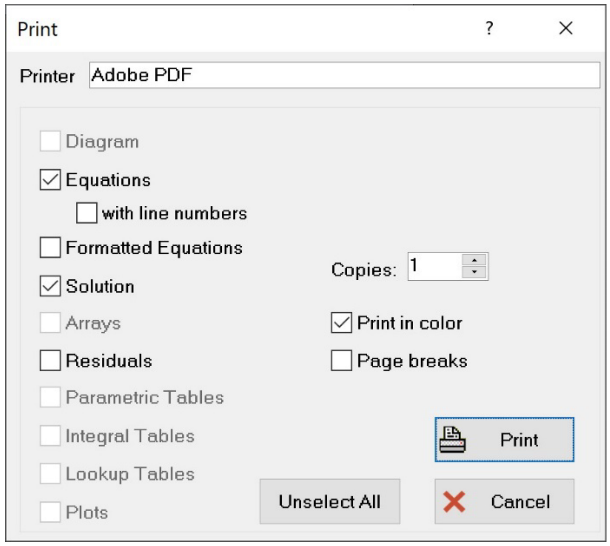


Figure 1.7. Print Dialog allows you to select which windows of the solution to print as well as the printer to use.

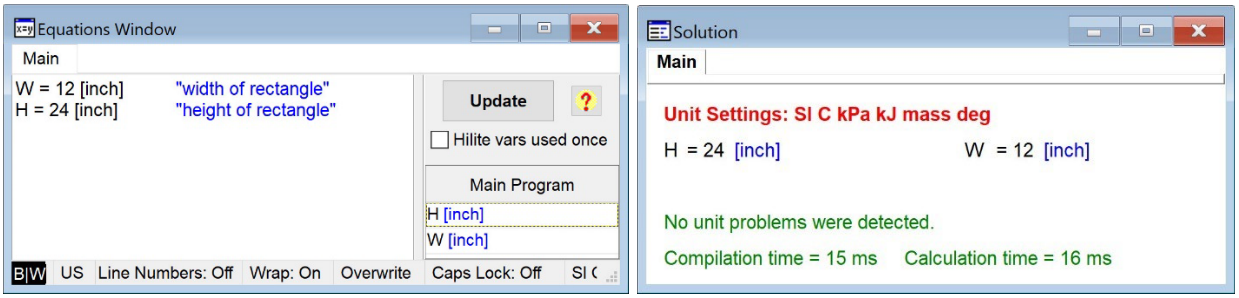
The printer can be selected, including the use of a pdf printer. Note that, by default, not all of the windows in your EES file will be printed. You need to select from among those shown along the left side of the dialog.

1.2E Units

Variables in engineering problems typically represent physical quantities and therefore they will have units. EES allows you to assign not only a value to each variable, but also its units. This is an incredibly useful feature of EES because the unit consistency of the equations is examined by EES and any unit conversion errors are flagged. Further, EES facilitates the process of converting between units by including a large number of built-in unit conversion factors. Finally, EES can display variables in dual units.

Setting, Converting, and Checking Units

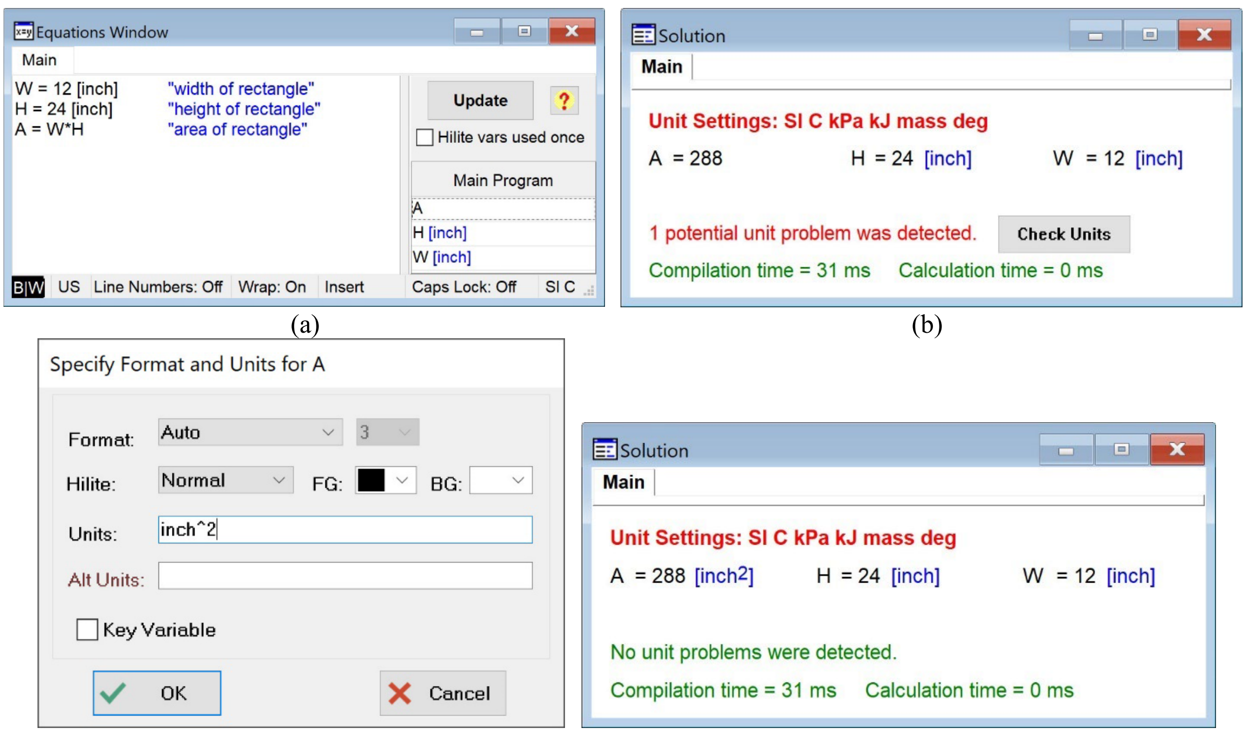
There are several ways to set the units of each variable. The units of a numerical constant (e.g., the number 2.5) can be set directly in the Equations Window by placing the unit within square brackets directly after the value. Figure 1.7(a) shows the Equations Window for an EES code that will compute the area of a rectangle of width *W* and height *H*. Note that the units of these variables (W and H) are set as they are entered, immediately after the number using square brackets. The resulting Solutions Window is shown in Figure 1.7(b); both the value and the units for each variable are indicated.



(a) (b)

Figure 1.8. (a) The Equations Window for an EES program that will be used to calculate the area of a rectangle. (b) The Solutions Window showing that the variables have both values and units.

It is not possible to set the units of a variable directly from the Equations Window. For example, in Figure 1.8(a) the equation needed to compute the area of the rectangle has been added. There is no way to specify the units of the new variable A directly in the same line of the Equations Window where the variable is calculated.



(a) (b) (c) (d)

Figure 1.9. (a) The Equations Window with the equation computing the area of the rectangle added. (b) The Solution Window with the units for the variable *A* undefined. (c) The Specify Format and Units Dialog that is accessed by right-clicking on the variable *A*. (d) The Solution Window with the units for the variable *A* defined.

The Solution Window for this situation is shown in Figure 1.8(b). Notice that the units for the variable A are not set and, as a result, EES provides the warning that 1 potential unit problem was detected. One way to set the units of the variable A is to right click on it in the Solutions Window which will bring up the Specify Format and Units dialog for that variable, shown in Figure 1.8(c). Enter the appropriate units in the Units edit box (in this case inch^2) and select OK to return to the Solutions Window, shown in Figure 1.8(d).

The units that EES recognizes can be seen by selecting Unit Conversion Info from the Options menu in order to bring up the Unit Conversion Information Dialog, shown in Figure 1.9. The left list box lists each dimension (e.g., length) while the right list box lists the units that are recognized by EES for the selected dimension (e.g., inch and ft). If you select two units in the right list box, then EES will display the unit conversion in red at the bottom of the dialog, as shown in Figure 1.9 for inch and ft.

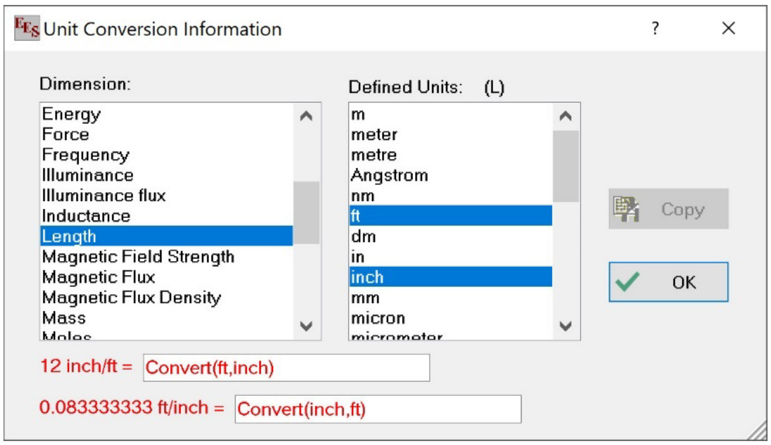


Figure 1.10. The Unit Conversion Information Dialog lists the units recognized by EES (in the right list box) organized by dimension (in the left list box). Selecting two units will display the relevant unit conversion.

The unit conversion information that is built into EES is easily accessible using the Convert function. The **Convert** function simply retrieves the conversion factor between any two units with the same dimension and therefore the function requires two arguments; the first argument is the unit to convert *from* while the second is the unit to convert *to*. For example, the EES statement:

```
L = 12 [mile]*Convert(mile,ft)
```

will determine the number of ft (63,360) that are in 12 miles. The unit conversion is easy and transparent to the user. The **Convert** function returned the correct unit conversion multiplier for the unit conversion that is specified by the two arguments (in this case 5280 ft/mile).

By default EES will check units anytime the equations are compiled or solved. You can also check units at any time by selecting Check Units from the Calculate menu (or using the shortcut key F8). This will bring up the Check Units Window which lists any unit problems that were detected. As a simple example, let's return to our program that calculates the area of a rectangle but this time enter the dimension for *W* in inch and for *H* in m, as shown in Figure 1.10(a). Selecting Solve from the Calculate menu will bring up the Solutions Window shown in Figure 1.10(b) which shows that there is 1 potential unit problem detected. Press the Check Units button to bring up the Check Units Window shown in Figure 1.10(c) which lists the equation that is not unit consistent. The equation for area has variables with units $inch^2$ on the left side (A) and inch-m (W * H) on the right; it is not unit consistent which indicates that a mistake has been made in the calculation.

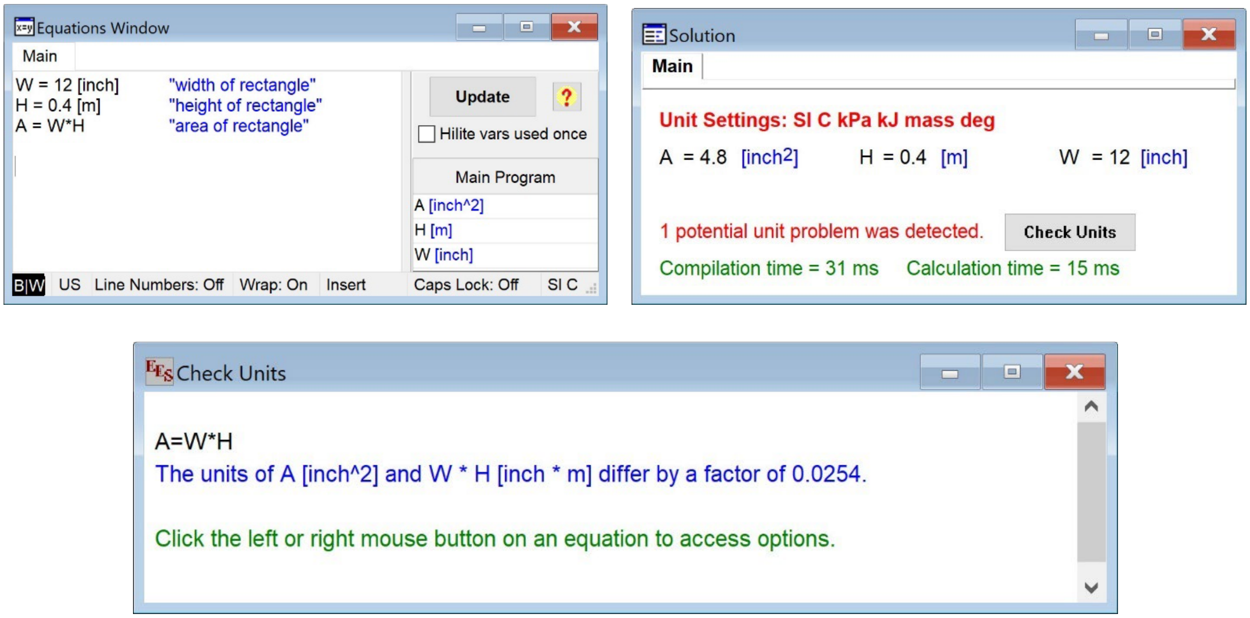


Figure 1.11. (a) Equations Window showing EES code to calculate the area of a rectangle with dimensions entered in inch and m. (b) Solutions Window showing that there is 1 potential unit problem. (c) Check Units Window accessed by selecting the Check Units button which shows the equation that is not unit consistent.

EXAMPLE E1.2 *The Characteristics of a Cylinder*

Calculate the surface area (cm^2), volume (liter), and mass (kg) of the cylinder shown in Figure 1 with height H = 12.7 inch and diameter D = 4.8 inch. The density of the cylinder material is ρ = 1120 kg/m^3.

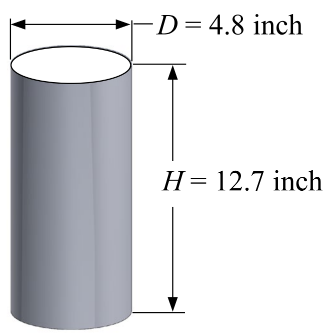


Figure 1. Cylinder.

SOLUTION

Road Map The dimensions of the cylinder will allow us to compute its geometric characteristics. The material density can be used to determine the mass. EES will be used to convert and check units as needed.

Governing Equations The surface area of the cylinder includes the sides and both ends according to

$$A_s = 2\frac{\pi D^2}{4} + \pi D H. \tag{1}$$

The volume of the cylinder is given by

$$V = \frac{\pi D^2}{4} H, \tag{2}$$

and the mass is the product of volume and density

$$m = \rho V \tag{3}$$

Equations (1) through (3) are three equations in the three unknowns A_s, V, and L.

Computation The inputs (D, L, and ρ) are entered in EES together with their units.

```
H = 12.7 [inch]                         "Height of cylinder"
D = 4.8 [inch]                          "Diameter of cylinder"
rho = 1120 [kg/m^3]                     "Density of cylinder material"
```

Equation (1) is used to compute the surface area. Note the use of pi# in the EES equation below, which refers to the built-in EES constant π, one of many such built in constants that are discussed in Section 1.4. Because the surface area is requested in cm^2, the **Convert** function is used to convert from inch^2 to cm^2.

```
A_s = (2*pi#*D^2/4+pi#*D*H)*Convert(inch^2,cm^2)     "Surface area"
```

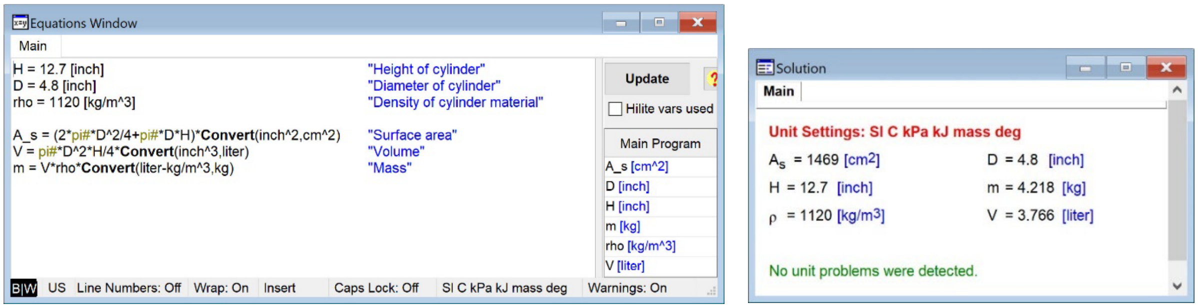
The volume is obtained using Eq. (2) and converted from inch^3 to liter using the **Convert** function.

```
V = pi#*D^2*H/4*Convert(inch^3,liter)                "Volume"
```

Finally, Eq. (3) is used to compute the mass. The units are converted from liter-kg/m^3 to kg using the **Convert** function.

```
m = V*rho*Convert(liter-kg/m^3,kg)                   "Mass"
```

The final Equations Window and Solutions Window are shown in Figure 2. Note in Figure 2(b) that the units for each of the calculated variables (A_s, V, and m) have been assigned in the Solutions Window and that EES has checked the units of each equation and not detected any unit problems.



(a) (b)

Figure 2. (a) Equations Window and (b) Solution Window.

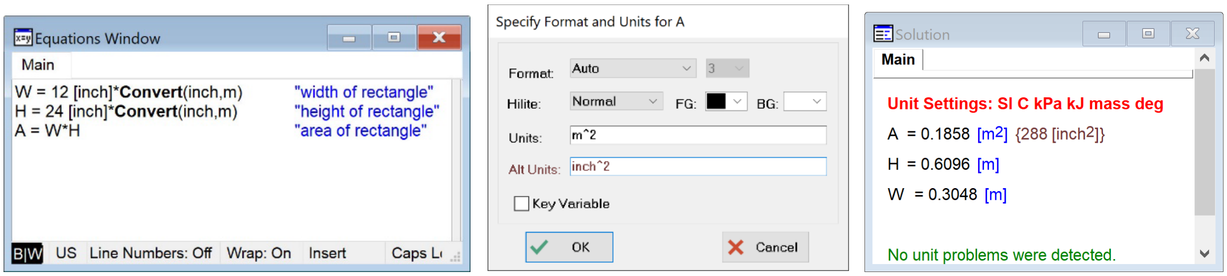
Discussion & Verification The solution to Eqs. (1) through (3) by hand would be easy but keeping track of the units and unit conversions might become tedious. The use of the **Convert** function in EES as well as its ability to check the units of the variables and equations reduces the opportunity to make unit errors. The next section discusses the use of base SI units for problem solving which further reduces the opportunity for making unit errors.

Base SI Units and Alternate Units

Every engineer should have his or her own method for dealing with units and unit conversions. Unit conversion errors are one of the most common mistakes that engineers make and the unit checking and unit conversion features of EES will help to avoid these problems. One common procedure for solving engineering problems that makes unit consistency easy to maintain is to use base SI units for all calculations. Inputs to an engineering problem are often encountered in arbitrary and mixed units. The first step in solving a problem is therefore to convert all inputs to the base SI system (i.e., kg, m, s, K, N, etc.). The calculations required to solve the problem are carried out using the base SI system and unit checking is rigorously applied in order to establish the dimensional consistency of each equation. The results of the calculations can be converted from the base SI system into whatever units are requested or are logical.

This procedure is convenient because the units of each variable are self-consistent in the SI unit system and therefore it is not necessary to constantly worry about applying the correct unit conversion to each equation during the development of a model. If you take this approach and you check the units of your equations, then you are actually verifying their dimensional (as well as their unit) consistency.

In order to facilitate reporting results in various units EES allows you to set an alternate unit for each variable as well as the primary unit that is used in the unit checking process. Notice in Figure 1.9(c) that the Specify Format and Units Dialog has an edit box both for units as well as alternate units. If an alternate set of units is provided for a variable, then it must have the same dimensions as the primary set of units. In this case, EES will display the variable in the Solution Window in both sets of units. As a simple example, Figure 1.12 shows the process of calculating the area of a rectangle by first converting the inputs (W and H) to base SI units and then using alternate units to display the output (A) in inch^2.



(a) (b) (c)

Figure 1.12. (a) The Equations Window with the equation computing the area of the rectangle using base SI units. (b) The Specify Format and Units Dialog that is accessed by right-clicking on the variable A showing that alternate units have been set. (c) The Solution Window showing that A is now displayed in both its actual units (m^2) as well as the alternate units ($inch^2$).

1.3E Functions

EES provides many built-in functions for basic mathematical and trigonometric capability. The built-in math functions can be conveniently accessed by selecting Function Information from the Options menu, which displays the Function Information dialog as shown in Figure 1.13.

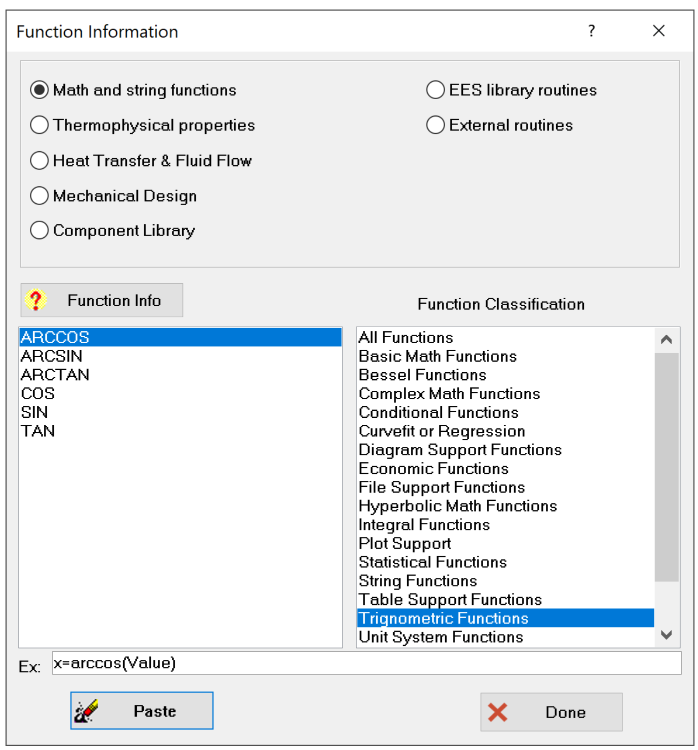
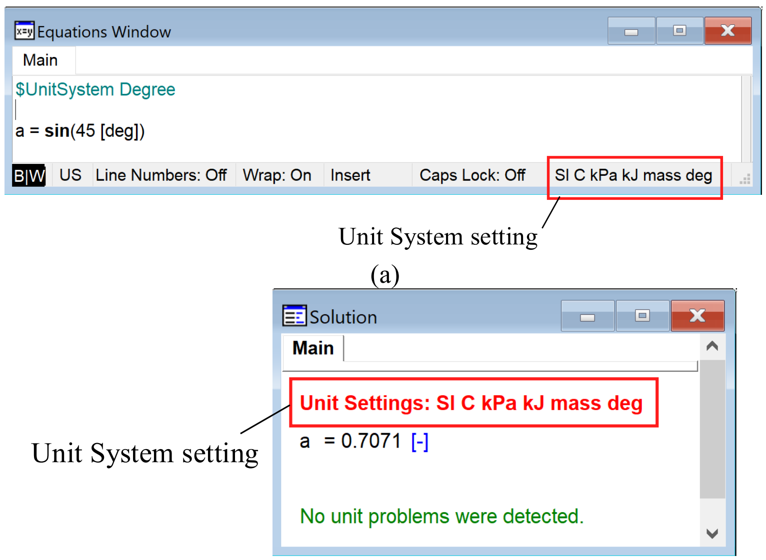


Figure 1.13. Function Information Dialog with Trigonometric Functions selected.

In a Statics course you will be primarily interested in trigonometric functions, which is the category selected in Figure 1.13. Depending on the problem, you may want these functions to interpret the input angles as being in either units of degrees or radians. EES' trigonometric functions can operate in either way but you must specify the units by setting the Unit System. This can be done in several ways, but the easiest is to place a $UnitSystem directive at the top of your EES code and follow it with either Degree or Radian, as shown in Figure 1.14.



(b)

Figure 1.14. (a) Equations Window with the $UnitSystem directive used to set the Unit System to degrees. (b) Solution Window showing the result of the calculation of sine. Notice in the Equations Window that the Unit System that has been specified is shown in the lower status bar. The Unit System is also shown at the top of the Solution Window.

1.4E Constants

There are a few constants that are useful to engineers. An obvious one is Pi (3.1416…). Another one is the universal gravitational constant that you learned about in Chapter 1 of the text. Constants in EES are denoted by a # sign after the name; constants are used like any other variable in EES except that their values cannot be changed in the Equations Window. A large number of common physical constants are pre-assigned in EES. These entries can be viewed by selecting Constants from the Options menu in order to access the Constants Dialog, shown in Figure 1.15.

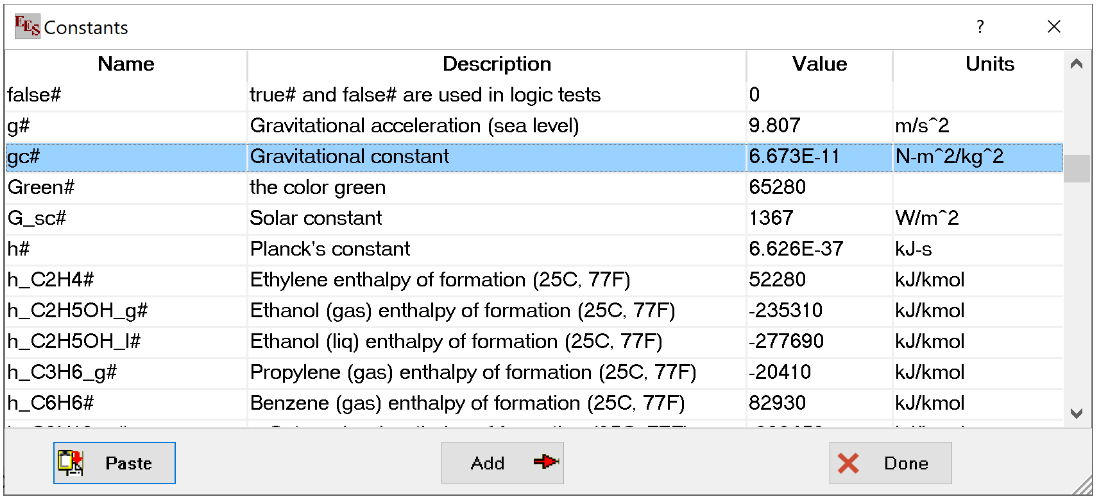
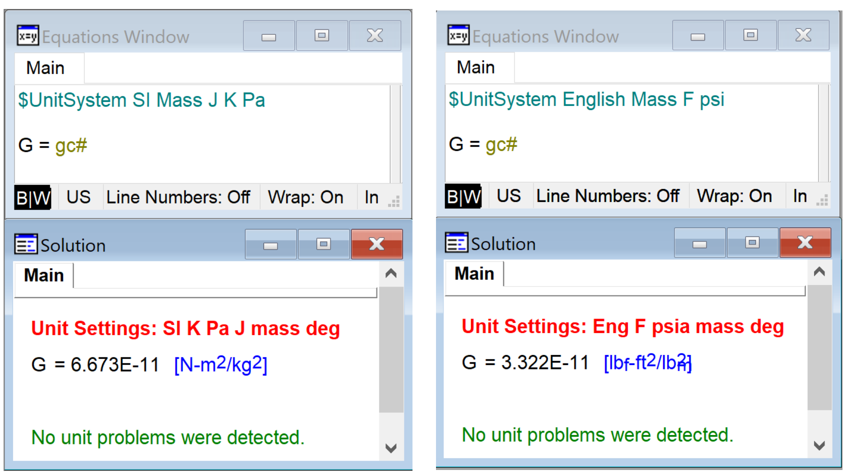


Figure 1.15. Constants Dialog.

Note that many constants are not dimensionless and therefore are associated with unit. For example, the gravitational constant gc# is equal to $6.673\text{x}10^{-11}$ N-m^2/kg^2. In non-SI units it would have a different value. Therefore when you are using constants you should be careful to set the unit system using the $UnitSystem directive, as discussed in Section 1.3E. Figure 1.16 shows that the value of a constant like gc# depends on the unit system setting.



(a) (b)

Figure 1.16. The value of gc# that is returned by the constant is different depending on whether the unit system is set to (a) SI or (b) English.

EXAMPLE E1.3 *Force on a Satellite*

A satellite is orbiting earth at an altitude $a = 22\text{x}10^3$ miles from the surface of the earth. The mass of the satellite is $m_s = 1500$ kg.

(a) Determine the force of gravity on the satellite (N).

(b) Determine the orbital altitude (in miles) at which the force increases to twice the value calculated in (a).

SOLUTION

Road Map The force to be determined is the gravitational force on the satellite which can be obtained using Eq. (1.10) of the text. The mass of earth and its radius will be taken to be m_{Earth} = $5.9736\text{x}10^{24}$ kg and $r_{\text{Earth}} = 6.371\text{x}10^{6}$ m, respectively.

Governing Equations The gravitational force between two objects can be computed using Eq. (1.10) of the text, repeated below

$$F = G\frac{m_1 m_2}{r^2}. \tag{1}$$

Here the masses are the mass of earth and the mass of the satellite and the distance between the objects, r, is the sum of the radius of the earth and the orbital altitude

$$F = G\frac{m_{\text{Earth}} m_s}{\left(a + r_{Earth}\right)^2}. \tag{2}$$

For part (a), Eq. (2) is a single equation in the single unknown F. For part (b) the force can be computed and so Eq. (2) becomes a single equation in the single unknown a.

Computation The unit system is specified using the $UnitSystem directive so that the gravitational constant is provided in base SI units. The inputs are entered in EES and converted to base SI units as necessary.

```
$UnitSystem SI Radian
a = 22000 [mile]*Convert(mile,m)          "altitude"
m_s = 1500 [kg]                           "satellite mass"
r_Earth = 6.371e6 [m]                     "mean radius of the earth"
m_Earth = 5.9736e24 [kg]                  "mass of the earth"
```

Part (a)

Equation (2) is used to compute the force on the satellite. Note the use of the built in constant gc# which corresponds to the universal gravitational constant G in Eq. (2).

```
"Part a"
F = gc#*m_Earth*m_s/(a+r_Earth)^2         "force on satellite"
```

Solving provides F = 342.7 N. Note that because the unit system was set to SI the universal gravitation constant, gc#, was provided in base SI units ($6.674\text{x}10^{-11}$ N-m^2/kg^2). All of the inputs were also converted to base SI units leading to the calculation of the force in base SI units (N) without the need for any unit conversions.

Part (b)

In order to do part (b) we need to compute a new force, F_b, that is twice the value of the force from part (a). Because EES is an equation solver it is not necessary to do any algebra to explicitly solve for a in Eq. (2). Rather, we can just input Eq. (2) again, this time using F_b and a_b to indicate the force and altitude.

```
"Part b"
F_b = 2*F
F_b = gc#*m_Earth*m_s/(a_b+r_Earth)^2          "new altitude"
```

Solving leads to the calculation of the new altitude, a_b, in base SI units: $a_b = 2317 \text{x} 10^7$ m. The problem statement requested a_b in miles so we will assign it alternate units of mile, leading to $a_b = 14.40 \text{x} 10^3$ mile.

The final Equations Window and Main tab of the Solutions Window are shown in Figure 1. For clarity of presentation, the two requested answers (F and a_b) are designated as key variables and the Key Variables tab of the Solutions Window is shown in Figure 1(c).

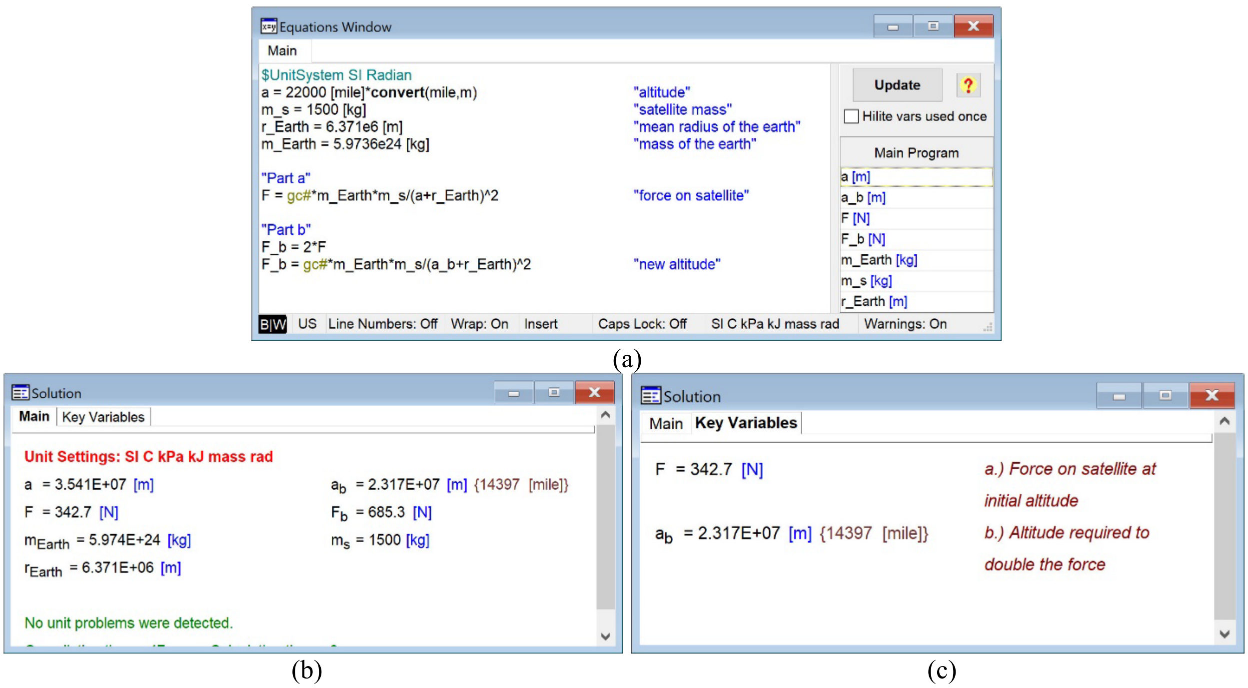


Figure 2. (a) Equations Window, (b) Main tab of the Solution Window, and (c) Key Variables Tab of the Solutions Window.

Discussion & Verification The satellite orbit had to decrease substantially in order to double the force of gravity. This result makes sense as gravity tends to increase as the distance between objects is reduced.